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MARKOVIAN QUEUEING SYSTEMS WITH
STOCHASTICALLY VARYING ARRIVAL RATES

by

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## 1. INTRODUCTION

Though assumed to be time homogenous in most studies of queueing models, arrival processes are seldom such when real systems are examined. In addition to the difficulties involved in the mathematical analysis of queueing systems having non-homogenous arrival processes, it appears that variations in these processes are generally slow, relatively to the queueing process itself, thus time intervals, in which arrival rate may be considered constant, are long enough to allow steady state analysis. However, often this is not the case, and non-homogeneity of the arrival process must be assumed. Queueing models with non-homogenous input have attracted the attention of queueing theorists starting in the mid-fifties with the works of Clarke [2] and Luchak [8]. Since then there is a thin flow of studies of such models, in the evergrowing stream of literature of queueing theory (see references [1] to [10]).

Moore [9] divides the approaches to "non-stationary" queueing systems into two primary categories: those considering the parameters of the arrival (and service) process (processes) to be continuous functions of time, and those assuming parameters to be step functions. Yet, there are still other possibilities. Many systems are characterized by the random nature of the variations of the basic processes. Hasofer [3] describes a situation where arrivals come from a large number of $N$ of "independent sources", each having a Poisson output with parameters $\lambda$, hence arrivals are of Poisson type of parameter $N\lambda$. If the number of sources is assumed to be a function of time $N(t)$, the arrival process will be a non-homogenous Poisson process with time dependent parameter

$\lambda N(t)$. Notice that in this presentation, $N(t)$ is assumed to be deterministic.

In many real life systems, however, the variations in the number of sources are due to stochastic phenomenae, thus modeling of such systems calls for the replacement of the (deterministic) function $N(t)$ by a stochastic process $\{N(t), t \geq 0\}$. Obvious candidates are those describing the behavior of populations (Poisson process, birth and death process, etc.). Restricting ourselves to Markov processes, we represent the arrival rates by a Markov jump process $\{\Lambda(t), t \geq 0\}$, on the parameters states set $\{\lambda_k\}$, with transitions $\lambda_0 \to \lambda_1 \to \lambda_2$ etc. The input parameter process is discussed in Section 3.

It should be noted that step functions are often used to approximate continuous functions, with proper selection of jump points and values. Similarly, certain (deterministic) arrival rate functions may be approximated by stochastic processes of the type considered in the present study, with proper selection of the parameters state set and transition rates.

We consider here Markovian queueing models. For reasons of simplicity we concentrate on the M/M/1 model. The results may be however, easily extended to more general classes of Markovian models, defining birth and death processes with stochastically dependent birth intensities and state dependent death intensities. For example, an M/M/r model will be specified by the arrival rate process $\{\Lambda(t), t \geq 0\}$ and service rate function $\mu(x) = \min\{\mu x/r, \mu\}$, when $x$ is the current queue size.

Queueing systems with non-homogenous input, may never attain stationarity, thus steady state solutions cannot be used. On the other hand, time dependent solutions may not yield measures of performance which are

essential if solutions are to be used for decision making. In such cases, either expected accumulated costs (for finite horizon only), or expected accumulated discounted costs should be considered. Accumulated costs, associated with decision processes will be discussed in a following communication. In the present paper we consider discounted costs.

Some of the results are based on previous results by Yadin and Zacks [11] and [12], and others, the summary of which is given in Section 2.

The main results are presented in Section 4.

## 2. PRELIMINARIES

A wide class of Markovian queueing models possess the following properties:

(a) input and output processes are both Markovian;

(b) actual service rate (output intensity, death intensity) is uniquely determined by the current queue size. It cannot exceed a given bound called the service capacity.

Considering discounting, a model is specified by:

(a) a set of parameters $\{\lambda,\mu,p\}$, where $\lambda$, $\lambda > 0$ is the arrival rate; $\mu$, $\mu > 0$, the service capacity; and $p$, $p > 0$ the discounting rate.

(b) an output function $\mu(x)$, $0 \leq \mu(x) \leq \mu$, representing output intensities as a function of the queue size $x$, $x = 0,1,\ldots$ .

For example, the output function of the M/M/r model is

$$\mu(x) = \min\{\mu x/r \ , \ \mu\} \ . \tag{2.1}$$

The original set of parameters can be replaced by $\{\lambda,\xi,\psi\}$, where

(2.2) $$\xi = \frac{1}{2\lambda}[\lambda + \mu + p - ((\lambda + \mu + p)^2 - 4\lambda\mu)^{1/2}],$$

$$\psi = \frac{1}{2\lambda}[\lambda + \mu + p + ((\lambda + \mu + p)^2 - 4\lambda\mu)^{1/2}] .$$

It is well known that

(2.3) $$\mu = \lambda \xi \psi$$

$$p = \lambda(1-\xi)(\psi-1) .$$

Consider a queueing process $\{X(t), t \geq 0\}$, $X(t)$ designating the queue size at epoch $t$, with a transition function

(2.4) $$P_{xy}(t) = P\{X(t) = y | X(0) = x\} , \quad x,y = 0,1,\ldots, ,$$

depending upon $\{\lambda,\mu,p\}$, (or alteratively $\{\lambda,\xi,\psi\}$) and $\mu(x)$. We assume that there exists a number $x_0$ such that $\mu(x) = \mu$ for all $x \geq x_0$.

For any $\beta$, $\beta \geq 0$, let $\Theta_\beta$ denote an independent, exponentially distributed random variable with mean $1/\beta$.

Let

(2.5) $$U^\beta m(x) = \int_0^\infty \sum_{y=0}^{\infty} m(y)\, p_{xy}(t)\, \beta e^{-\beta t}\, dt$$

$$= E[m(X(\Theta_\beta)) | X(0) = x] ,$$

be defined for any function $m(x)$, $x = 0,1,\ldots$ such that the convergence condition

(2.6) $$\lim_{x\to 0} \psi^{-x}\, U^\beta m(x) = 0$$

is satisfied. $U^\beta m(x)$ defined in (2.5) is known as the $\beta$-potential of the charge $\beta m(x)$.

It is easily seen that $U^{\beta}$ is a linear operator, satisfying

$$(2.7) \qquad U^{\beta}[a_0 + \sum_{i=1}^{n} a_i \, m_i(x)] = a_0 + \sum_{i=1}^{n} a_i \, U^{\beta} \, m_i(x) \quad ,$$

for any set of coefficients $\{a_0,\ldots,a_n\}$ and charges $\{\beta m_1(x),\ldots,\beta m_n(x)\}$, such that $U^{\beta} m_j(x)$, $j = 1,\ldots,n$, exist.

Let

$$(2.8) \qquad G_x(z,p) = U^p \, \frac{1}{p} \, z^x \quad , \quad |z| < \psi \quad ,$$

be the Laplace transform of the generating function of the transition distribution $\{p_{xy}(t)\}$ of the queueing process $\{X(t),\ t \geq 0\}$, evaluated at $p$. It can be shown that for $|z| < \psi$ ($\psi$ be a function of the discounting rate $p$) the convergences conditions are satisfied. The transform $G_x(z,p)$, which is given for a variety of models, may conveniently yield $U^p m(x)$ for a wide class of charges, in the same fashion as generating functions generate probabilities and moments. Indeed, if $(x)_r = x(x-1)\cdots(x-r+1)$ then

$$(2.9) \qquad U^p \, \frac{1}{p}(x)_r \, z^{x-r} = \frac{\partial^r}{\partial z^r} \, G_x(z,p) \quad , \quad |z| < \psi \quad ,$$

$$(2.10) \qquad U^p \, \frac{r!}{p} \, \delta_{r,x} = \frac{\partial^r}{\partial z^r} \, G_x(z,p)\Big|_{z=0} \quad ,$$

where $\delta_{r,y} = 1$ , $r = y$; $\delta_{r,y} = 0$ otherwise. In particular,

$$(2.11) \qquad U^p \, \frac{1}{p}(x)_r = \frac{\partial^r}{\partial z^r} \, G_x(z,p)\Big|_{z=1} \quad .$$

Clearly, computations based on the last results are practical for small integers $r$ only. Combining these results with (2.7) we obtain an efficient tool for the evaluation of $U^{\beta} m(x)$ for various charges.

Further analysis requires the consideration of specific models. For reasons of simplicity we shall concentrate on the M/M/1 model. Extensions to other models are immediate.

It is well known that for the M/M/1 model

$$(2.12)\qquad U^p \frac{1}{p} z^x = G_x(z,p) = \frac{(1-\xi)\, z^{x+1} - (1-z)\, \xi^{x+1}}{\lambda(1-\xi)(z-\xi)(\psi-z)} \quad , \quad |z| < \psi \ .$$

particularly

$$(2.13)\qquad U^p \frac{1}{p} \xi^x = \frac{(1-\xi)\, x + 1}{\lambda(1-\xi)(\psi-\xi)}\ \xi^x \ .$$

It follows from (2.7), (2.11) and (2.12) that

$$(2.14)\qquad U^p[s + qx] = \frac{1}{p}\left[ s + q\left( x + \frac{\lambda-\mu}{p} + \frac{\xi^{x+1}}{1-\xi}\right)\right] ,$$

as shown by Yadin and Zacks [11].

We are considering in the present study the conditional expectations of accumulated discounted costs. Let $M(x)$ denote such expectation evaluated at $t$, $t \geq 0$, given $X(t) = x$. Because of the Markovian nature of the model this expectation is independent of $t$. Assuming that the costs are generated by a total charge of $pm(x)$ per time unit, we have

$$(2.15)\qquad M(x) = U^p\, m(x) \ .$$

The charge $pm(x)$, consists of holding costs of $x$ customers and other possible costs. The costs function $m(x)$ represents the accumulated discounted costs associated with the residence of $x$ customers for an infinitely long period.

Obviously, we shall limit ourselves to costs functions such that $M(x)$ is finite for any finite $x$. Furthermore, we shall assume that $m(x)$ is such that the convergence condition (2.6) is satisfied. Linear

functions, polynomials and exponential functions $\frac{1}{p} z^x$ such that $|z| < \psi$, are examples of such $m(x)$ functions.

The process $\{X(t), t \geq 0\}$ is a birth and death process with birth intensity $\lambda$ and death intensities $\mu(x)$. Transition probabilities are $\lambda[\lambda + \mu(x)]^{-1}$ and $\mu(x)[\lambda + \mu(x)]^{-1}$, respectively. Let $\Theta$ denote the random time of the first event (either birth or death). $\Theta$ has an exponential distribution with parameter $\lambda + \mu(x)$. It follows that

(2.16)
$$M(x) = E\left\{ \int_0^{\Theta} pm(x)\, e^{-pt}\, dt + \right.$$
$$\left. + \exp(-p\Theta)\left[\frac{\lambda}{\lambda+\mu(x)} M(x+1) + \frac{\mu(x)}{\lambda+\mu(x)} M(x-1)\right]\right\}$$

Integration of (2.16) yields the following functional equation for $M(x)$:

(2.17)
$$M(x) = \frac{\lambda\, M(x+1) + \mu(x)\, M(x-1) + p\, m(x)}{\lambda + \mu(x) + p} .$$

The unique solution of the functional equation (2.17) which satisfies the convergence solution is the one presented at (2.15). Suppose there exist two distinct solutions, $M(x) = U^{\beta} m(x)$ and $M(x) + d(x)$. Clearly, the difference $d(x)$ satisfies the equation.

(2.18)
$$d(x) = \frac{\lambda d(x+1) + \mu(x)\, d(x-1)}{\lambda + \mu(x) + p} , \quad \text{for all } x.$$

Hence, for all $x \geq x_0$ (for which $\mu(x) = \mu$)

(2.19)
$$d(x) = a\, \xi^{x-x_0} + b\, \psi^{x-x_0} .$$

In order to satisfy (2.6) we must set $b = 0$.

Substitution for $M^*(x)$ and $M^*(x) + a\, \xi^{x-x_0}$ into (2.17) (for $x \geq 0$) yields

(2.20)
$$a[\lambda\xi(1-\xi) - \mu(1-\xi) + p] = ap(1-\xi) = 0 .$$

Since $p > 0$ and $\xi < 1$ we must have $a = 0$. Thus $d(x) = 0$ for all

$x \geq x_0$ and therefore for all $x$.

$M(x)$ can be approximated by a sequence $\{M^{(k)}(x)\}$ such that

$$(2.21) \qquad M^{(k)}(x) = \frac{\lambda M^{(k-1)}(x+1) + \mu(x)\, M^{(k-1)}(x-1) + pm(x)}{\lambda + \mu(x) + p}, \quad k \geq 1 .$$

It can be shown that $\{M^k(x)\}$ uniformly converges to the solution of (2.17), for any $M^{(0)}(x)$ such that the convergence conditions are not violated.

## 3. THE MAIN RESULTS

Let $\alpha_0, \alpha_1, \ldots$ and $\lambda_0, \lambda_1, \ldots$ be two sequences of non-negative parameters, and let

$$(3.1) \qquad \begin{aligned} \tau_0 &= 0 \\ \tau_k &= \tau_{k-1} + T_{k-1}, \quad k = 1,2,\ldots , \end{aligned}$$

where $T_0, T_1, \ldots$ are mutually independent, exponentially distributed random variables such that $E[T_j] = \alpha_j$, $j = 0,1,\ldots$ .

Define a Markov process $\{\Lambda(t), t \geq 0\}$, such that

$$(3.2) \qquad \Lambda(t) = \lambda_k, \quad \tau_k \leq t < \tau_{k+1}, \quad k = 0,1,2,\ldots .$$

By setting $\alpha_n = 0$ for some $n$ we mean that $\Lambda(t) = \lambda_n$ for $\tau_n \leq t < \infty$, in other words, the last change in $\Lambda(t)$ is at $\tau_n$.

Consider a non-homogenous Poisson process $\{A(t), t \geq 0\}$ with intensity $\Lambda(t)$ at $t$, $t \geq 0$. The objective of the present work is the evaluation of the expectations of the accumulated discounted costs of queueing systems of the class presented in Section 2, with the non-homogenous arrival process $\{A(t), t \geq 0\}$. Note that the arrival process

is completely defined by $\underline{\alpha} = \{\alpha_k\}$ and $\underline{\lambda} = \{\lambda_k\}$. Hence, the model is specified by the set $\{\underline{\alpha}, \underline{\lambda}, \mu, p\}$ of parameters; by the service function $\mu(x)$ and by the costs function $m(x)$.

Denote by $M_k(x)$ the conditional expectation of the accumulated discounted costs evaluated at some epoch $t$, for infinite period, given $\tau_k \leq t < \tau_{k+1}$ and $X(t) = x$.

$$(3.3) \qquad M_k(x) = \int_t^{\infty} \sum_{y=0}^{\infty} p\, m(y)\, P\,\{X(\tau) = y \mid X(t) = x,\ \tau_k \leq t < \tau_{k+1}\} \cdot$$
$$\cdot\, e^{-p(\tau - t)} d\tau =$$
$$= E[m(X(t+\Theta_p)) \mid X(t) = x,\ \tau_k \leq t < \tau_{k+1}] \,.$$

Arguments, similar to those leading to (2.16) and (2.17) yield

$$(3.4) \qquad M_k(x) = E\left\{\int_0^{\Theta} p\, m(x)\, e^{-pt}\, dt + \right.$$
$$+\, e^{-p\Theta}\left[\frac{\alpha_k}{\lambda_k + \alpha_k + \mu(x)} M_k(x+1) + \right.$$
$$\left.\left. + \frac{\mu(x)}{\lambda_k + \alpha_k + \mu(x)} M_k(x-1) + \frac{\alpha_k}{\lambda_k + \alpha_k + \mu(x)} M_{k+1}(x)\right]\right\}.$$

where $\Theta$ has an exponential distribution with parameter $\lambda_k + \alpha_k + \mu(x)$. This expression is reduced then to

$$(3.5) \qquad M_k(x) = \frac{\lambda_k M_k(x+1) + \mu(x) M_k(x-1) + \alpha_k M_{k+1}(x) + pm(x)}{\lambda_k + \mu(x) + \alpha_k + p} \,.$$

Following the results of Section 2, we observe that the unique solution of (3.5) satisfying the convergence conditions, is

$$(3.6) \qquad M_k(x) = U^{\alpha_k + p}\, m_k(x) \quad ,$$

where

$$m_k(x) = \frac{\alpha_k M_{k+1}(x) + pm(x)}{\alpha_k + p} \quad . \tag{3.7}$$

Given the costs function $m(x)$ and $M_n(x)$ for some $n > 0$, $M_k(x)$ may be recursively determined for all $0 \leq k < n$, either numerically (applying approximating sequences generated by equations similar to (2.21) and based on (3.5)) or analytically.

Clearly, development of explicit formulae for $M_k(x)$ is limited to certain queueing models and costs function. We shall demonstrate the procedure by deriving the formulae for the M/M/1 model and linear costs. The procedure is applicable however to other cases.

Consider the M/M/1 model, specified by the set of parameters $\{\underline{\alpha}_k, \underline{\lambda}_0, \mu, p\}$; the service rate function $\mu(x) = \max\{x\mu, \mu\}$ and cost function

$$m(x) = \frac{1}{p}[s + qx] \quad . \tag{3.8}$$

Let

$$\xi_j = \frac{1}{2\lambda_i}[\lambda_i + \alpha_i + \mu + p - ((\lambda_i + \alpha_i + \mu + p)^2 - 4\lambda_i\mu)^{1/2}] \; , \tag{3.9}$$

$$\psi_i = \frac{1}{2\lambda_i}[\lambda_i + \alpha_i + \mu + p + ((\lambda_i + \alpha_i + \mu + p)^2 - 4\lambda_i\mu)^{1/2}] \; ,$$

$$j = 0,1,2,\ldots \; .$$

The following procedure is limited to cases where $\xi_i \neq \xi_k$ for $i \neq k$. This limitation presents some complications in development. Other assumptions may lead to more complicated results.

Suppose we have for a given $k$, $0 \le k \le n$

$$M_k(x) = \frac{1}{p}\left[s + q\left(x + \frac{\eta_k - \mu}{p} + \sum_{j=k}^{n} C_{kj} \frac{\xi_j}{1-_j}\right)\right] . \tag{3.10}$$

for appropriate sequences of constants $\{C_{kj};\ k = 0,\ldots,n;\ j = k,\ldots,n\}$.

Then, following (3.7)

$$m_{k-1}(x) = \frac{1}{p}(s + qx) + \frac{\alpha_{k-1}}{\alpha_{k-1}+p} \cdot \frac{q}{p}\left[\frac{\eta_k - \mu}{p} + \sum_{j=k}^{n} C_{kj} \frac{\xi_j^{x+1}}{1-\xi}\right] . \tag{3.11}$$

Furthermore, according to (2.14)

$$U^{\alpha_{k-1}+p} \frac{1}{p}[s + q(x + \frac{\alpha_{k-1}}{\alpha_{k-1}+p} \cdot \frac{\eta_k - \mu}{p})] = \frac{1}{p}\left[s + q\left(x + \frac{\eta_{k-1} - \mu}{p} + \frac{\xi_{k-1}^{x+1}}{1-\xi_k}\right)\right] , \tag{3.12}$$

where

$$\eta_{k-1} = \frac{\alpha_{k-1}\,\eta_k + p\lambda_{k-1}}{\alpha_{k-1} + p} . \tag{3.13}$$

From this result and (2.12) we obtain that

$$U^{\alpha_{k-1}+p} \frac{\alpha_{k-1}}{\alpha_{k-1}+p} \cdot \frac{q}{p} \sum_{i=k}^{n} C_{ki} \frac{\xi_i^{x+1}}{1-\xi_i} = \tag{3.14}$$

$$= \frac{q}{p} \sum_{j=k}^{n} C_{k-1,j} \left(\frac{\xi_j^{x+1}}{1-\xi_j} - \frac{\xi_{k-1}^{x+1}}{1-\xi_{k-1}}\right) ,$$

where

$$C_{k-1,j} = \frac{\alpha_{k-1}\,\xi_j}{\lambda_{k-1}(\xi_j - \xi_{k-1})(\psi_{k-1}-\xi_j)} C_{kj} , \quad j = k,\ldots,n . \tag{3.15}$$

Moreover, since

$$\lambda_{k-1}(\xi_j - \xi_{k-1})(\psi_{k-1} - \xi_j) = \xi_i[(\lambda_{k-1} - \lambda_j)(1-\xi_j) + \alpha_{k-1} - \alpha_k] , \tag{3.16}$$

we obtain

$$C_{k-1,j} = \frac{\alpha_{k-1}}{(\lambda_{k-1}-\lambda_j)(1-\xi_j) + \alpha_{k-1} - \alpha_j} C_{k,j} , \quad j = k,\ldots,n . \tag{3.17}$$

Finally, introduce the constants

$$C_{k-1,k-1} = 1 - \sum_{i=k}^{n} C_{k-1,i} \, , \tag{3.18}$$

and combine the partial results obtained above then

$$M_{k-1}(x) = U^{\alpha_{k-1}+p} m_{k-1}(x) = \tag{3.19}$$

$$= \frac{1}{p}\left[ s + q\left( x + \frac{\eta_{k-1}-\mu}{p} + \sum_{j=k-1}^{n} C_{k-1,j} \frac{\xi_j^{x+1}}{1-\xi_j} \right)\right] .$$

Suppose we assume that $\alpha_n = 0$ for a given $n$, which means that $\Lambda(t) = \lambda_n$ for all $\tau_n \leq t < \infty$, then

$$M_n(x) = \frac{1}{p}\left[ s + q\left( x + \frac{\lambda_n - \mu}{p} + \frac{\xi_n^{x+1}}{1-\xi_n} \right)\right] . \tag{3.20}$$

Commencing with $\eta_n = \lambda_n$ and $C_{n,n} = 1$ we apply (3.13), (3.17) and (3.18) to the recursive computation of the coefficient $\eta_k$, $C_{k,i}$, $i = k,\dots,n$, for all $0 < k < n$.

Suppose now that we have $\xi_{k-1} = \xi_j$ for some pair $j,k$ $j \geq k$. Formula (2.12) cannot be used then for the derivation of (3.14). Instead of (2.12) we have to apply (2.13) and introduce terms of the form $x\xi^x$. The introduction of such terms complicates the equations. Further development may be possible, but it requires the use of expression (2.9). The procedure described here may be proved unpractical in cases where the $\xi_i$'s are not distinct.

In some cases there is certain freedom in the selection of the sequences $\underline{\alpha}$ and $\underline{\lambda}$. This is the case where $\{\Lambda(t), t \geq 0\}$ is designed to approximate some given deterministic function $\lambda(t)$. In such cases, whenever it is necessary to assume that $\lambda_i = \lambda_k$ for $i \neq k$ it is still possible to

assume $\alpha_i \neq \alpha_k$ and have $\xi_i \neq \xi_k$. Distinct but close $\xi_i$'s may cause however computational problems.

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Queueing systems which ~~are represented by birth and death~~ processes with time dependent birth intensities and state dependent death intensities are studied. Birth intensities are described by a Markov jump processes. Explicit results are obtained for the M/M/1 model.

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